INDEX

A

A/B test
  click-through rate example (*see* click-through rate example)
  parameter estimation with, 194
  techniques, 150
  usage, 149–150
absolute deviation, 103–106
absolute error, 53
absolute value, 105–106. *See also* variance
alternate hypotheses, 9–10, 11
AND, 21–22, 24–25
antiderivatives, 227
averaging
  defining, 94
  mean, compared to, 99–100
  probability, relationship between, 95
  snowfall estimation example
    error minimization through averaging, 94–95
    errors, 97, 98–99
    extreme distribution, 97–98
    overview, 94
    simplifying, 95–97

B

Bayes, Thomas, 146
Bayesian A/B test. *See* A/B test
Bayesian Battlers game example, 41–43
Bayesian reasoning
  data, observing, 4
  falsifiability, 180–181
  overview, 3–4
  psychic rolling dice example
    alternate hypotheses, 178–179
    likelihoods, comparing, 176–177
    prior odds, including, 177–178
  Twilight Zone example
    Bayes factor, measuring, 169–170
    hypotheses regarding, 175, 176
    Mystic Seer, using Bayes factor to understand, 168–169, 170–171, 172
    overview, 168
    prior beliefs, accounting for, 170–171
    psychic powers, developing, 171–172
Bayes factor
  carnival game analysis, 184
    likelihood ratio, 188–189
    multiple hypotheses, 185, 186–188
  defining, 157
  formula, 159
  posterior odds, 160
    hypothesis testing, 161–162
    loaded die, testing, 161–162
    rare diseases example, 162–164
  prior beliefs, relationship between, 157
  prior odds, 159–160
Bayes' theorem
  beliefs, use in analyzing, 64–65, 75, 80–81
  calulating, in LEGO example (*see* LEGO visualization)
  climate change probability example, 64–65
  crime scene example
    alternate hypotheses, 78–79
    beliefs, strength of, 80, 81
    likelihood, solving for, 75, 78
    normalizing data, 76–77
    posterior probability, 76
    prior, calculating, 75–76, 78–79
    scene of crime, 74
    unnormalized posteriors, 80–81

Bayes' theorem (*continued*)
evidence observation, use in, 65
formula, 64, 67, 158
LEGO example (*see* LEGO visualization)
likelihood, 158 (*see also* likelihood)
posterior probabilities, 158 (*see also* posterior probabilities)
prior probability, 158 (*see also* prior probabilities)
proportional form, 87, 158
statistics, importance to, 64, 67
beliefs
data, relationship between, 10–11, 64–65, 74
distribution of, 84–85
irrational, 179
measuring, 18–19
mutability of, 11
origins of, 11
prior, 5, 86, 141, 143–144, 157, 170–171
probability distribution of, 49, 83
ranges of, 83
strength of, analyzing, 64–65, 67, 80–81
worldview, relationship between, 64–65
beta distributions
applications, 45
beliefs, of, 87
binomial distributions, *versus,* 50, 52
changes in, as information is gained, 138, 142
estimating with, 121
Gacha game example, use in, 54
mean of, 125
normal distributions, compared to, 121–122
normalizing values, 51
overview, 45
parameters, 84, 135, 140
prize distribution example, 191–194
probability density function of, 51–52, 124
probability, true, 121
quantiles, of, 135
undefined, 145
binomial coefficients, 36–38. *See also* combinatorics
binomial distributions
beta distributions, *versus,* 50, 52
examples, 34
outcomes, 34
overview, 34
parameters, 34
Probability Mass Function (PMF), relationship between, 39
probability, use in calulating, 47–48
shorthand notations, 34–35
solving for, 43
structure, 34–35

C

C-3PO example, 84–88
`c()`, 200
calculus, fundamental theorem of, 228
change, rate of, 223
click-through rate example, 149
A/B test
conversion rate, 150–152
data collection, 151
parameters, 151–152
prior probability, finding, 150
setting up the A/B test, 150
Monte Carlo simulations, 152–153, 153–154, 154–155
combinatorics, 16, 37–38
conditional probabilities, 5–6
beliefs, impact of, 6
color blindness example, 61–64
defining, 60, 65
experiences, impact of, 6
flu vaccine risks example, 60–61
likelihoods, 64
overview, 60
reversing, 62–63, 67–68
confidence intervals (CIs), 132–133
conspiracy theories, 181
continuous distributions, quantifying, 52–53, 55
conversion event, estimating. *See also* click-through rate example
beta distributions for, 138, 139–140
PDF, using, 124–127, 135
prior probabilities regarding, 141–142
conversion rate, 124
critical region, 132–133
`cumsum()`, 204

cumulative distribution function (CDF)
antiderivative of a PDF, 127
confidence intervals, estimating, 132–133
distribution, sums of the parts of, 127
interpreting, 130
intuitiveness, 128
inverse of, 133–134
mean of, 131
median of, 130, 133
quantile function, use in calculating, 133–134
R programming language, use in, 132
usefulness, 132
visualization of, 128, 130

D

data
Bayesian statistics, importance to, 63
beliefs, relationship between, 10, 11, 64–65, 74
high-probability, 7
hypothesis, relationship between, 6–10
normalizing, 76–77
observation of, 4, 7
probability, relationship between, 10, 76, 77
size of sets, 75
spread, measuring, 103–105, 108
`dbeta()`, 53, 126–127
derivative, 223–227
`dfunction()`, 126
`diff()`, 204
`dnorm()`, 126, 133

E

errors, statistical analysis, 84
error value, 105
evidence, observation of, 65
expectation, defining, 99. *See also* mean
exponential penalty, 107

F

factorials, 37
falsifiability, 180–181
Frequentist Fighters! game example, 53–54
`function`, 209
functions
calculus, use of, 216–217, 218–220
defining, 216
integrals, use of (*see* integrals)
fundamental theorem of calculus, 228

G

Gacha games
Bayesian Battlers game example, 41–43
Frequentist Fighters! game example, 53–54
probability distributions, 41–43
reverse-engineering, 53–54
`ggplot2`, 210

H

Han Solo example, 84, 86, 139
hypotheses
alternate, 9–10, 11, 78–79, 178–179
beliefs, relationship between, 74, 83
confidence in, 83
developing, 48
formal, 7
formulating, 4, 6–7
hidden, 180
infinite, 49–50, 55
multiple, 183, 185
probability, relationship between, 7, 17–18, 47, 48–49
R programming language, searching with (*see* R progamming language)
testing, 49

I

`ifelse`, 188, 205–206
independent probabilities, 59, 61
inference, 46–47, 48
infinity, 117
integrals, 51, 119
applications, 53
approximating, visually, 131–132
beta distribution of, 128
derivative, relationship between, 228
estimating with, 219–223
using R to solve for, 131
`integrate()` function, 53, 118
intuition, use of, 70–72, 107, 108, 128

K

Keynes, John Maynard, 146

L

Laplace, Simon-Pierre, 146
LEGO visualization
  Bayes' theorem, calulating, 71–72
  conditional probabilities, 69, 70
  intuition, use of, 70–72
  mutually exclusive events, 68
  physical representations, determining, 70
  probabilities, 68–69, 71
  ratios, 71
  reasoning, use of, 70
likelihood, 73
  beliefs, relationship between, 87
  C-3PO example, 85, 87
  defining, 74
  prior probabilities, including, 177
  solving for, 75, 78
likelihood ratio, 176–187
  priors, adding, 188–189
`lines()`, 212
logic, 14, 30
logical operators, 21. *See also* specific operators
  reasoning with, 22

M

Mandela effect, 16–19
`max()`, 187
mean
  beta distribution, of, 125
  cumulative distribution function (CDF), of, 131
  defining, 99
  estimating with, 111–112
  parameter in normal distribution, 111, 114
  representation of, 99–100
  usage, 100
mean absolute deviation (MAD), 104–108
median, 130, 133
Monte Carlo simulation, 149
  click-through rate example, use in, 152–153, 153–154, 154–155
  defining, 152
  power of, 154
μ, 99. *See also* mean

N

negative infinity, 117
neural networks, 7
noninformative prior, 144–145
normal distributions
  bell-shaped, 114
  beta distributions, compared to, 121–122
  defining, 114
  integrating functions, 117–118
  mean, 111, 114–116, 118–119
  parameters, 111, 114
  probability density function (PDF) of, 116, 118
  range of values, 111
  standard deviations, 111, 114, 116, 118
  villain bomb example, 112–114, 116–118, 120
NOT, 21–22
n sigma events, 120

O

observation, 11
  data, of, 4, 7
odds, in probability, 17
Ω, 15
OR, 21
  logical reasoning with, 22
  probabilities, combining with, 26
    example, calculating probability of fine, 29–30
    mutually exclusive events, calculating OR for, 26–27
    sum rule for non-mutually exclusive events, 28–29, 30

P

parameter estimation
  applications, 93
  averaging (*see* averaging)
  hypothesis testing with, 184–185
  prize distribution example, 191–194
parameters, 93. *See also* parameter estimation
percentile, defining, 123

permutations, 35. *See also* probability distributions
`plot()`, 212, 214
plotting, 210–213
point plot, 212
population variable, 63
posterior, normalized, 87
posterior distributions, 87
  beliefs, relationship between, 153
  beta distributions, use with, 140–142
posterior odds, 160, 170, 177–179, 188, 190
posterior probabilities, 73–74, 76–77
posteriors, ratio of. *See* ratio of posteriors
prior, noninformative, 144–145
prior odds, 159–160
prior probabilities, 73
  beliefs, relationship between, 141
  C-3PO example
    beliefs, C-3PO's, 84–88
    beta distribution, 87
    data, 84–85, 87–88
    overview, 83
  calculating, 75–76, 78–79
  controversy regarding, 83, 86
  conversion events, use in estimating, 139
  data regarding, 145, 146
  defining, 74
  distributions, 143
  finding, as part of an A/B test, 150
  Han Solo example, 84, 86, 139
  psychic die example, 178
  subjectivity of, 83
probability
  applications, 46
  beliefs, calculating as ratio of, 15, 19–20
  calculating, 47–50
  conditional (*see* conditional probabilities)
  data, relationship between, 10, 76–77
  data observation, as part of, 4
  defining, 14
  determining, 5
  distributions (*see* probability distributions)
  events, counting outcomes of, 14–15, 18
  high, data supporting, 8
  hypotheses, relationship between, 7, 47–49
  independent, 59, 61
  language of, 7, 8
  odds, determining, 17
  posterior, 73–74, 76–77
  prior (*see* prior probabilities)
  product rule of, 24–25, 61–62, 79
  quantifying, 3
  rules of, 43, 61–62
  solving for, 17–18
  statistics, *versus,* 46
  sum rule for non-mutually exclusive events, 28–30, 61
probability density function (PDF)
  antiderivative of, 127
  applications, 51–52
  C-3PO example, 85
  conversion event, use in estimating, 124
    beta distribution, 125–127
    interpeting, 125–126
    R, use of PDF in, 126–127
    visualizing, 125–126
  formula, 50–51
  integration, use in, 127
  normal distribution, of, 116–118
  parameters, 50
  plot, 51–52
probability distributions
  applications, 34
  beliefs, of, 49, 83
  beta distributions (*see* beta distributions)
  binomial (*see* binomial distributions)
  building, 190–191
  Gacha games example, 41–43 (*see also* Gacha games)
  generalizations, 41
  outcomes, 35–36, 38–39
  overview, 33–34
  permutations, 35
  simplifying, 35
probability mass function (PMF), 39, 50
probability theory, 4, 14
product rule of probability, 24–25, 61–62, 79

Q

quantile function, 133–134

R

R programming language
  CDF, use of in, 132
  doubles, 198
  functions, defining, 209–210
  installing, 196
  integrals, solving for, 131
  likelihood ratios, calulating, 188–189
  logicals, 199
  missing values, 200
  multiple hypotheses testing with, 186–188
  plots, generating, 210–213
  probability density dunction (PDF), use of, 126–127
  quantiles, calculating, 135
  random numbers generated by, 208–209
  R Script, 197
  strings, 199, 203
  vectors, 200–202
    `length()`, 202–203
    `sum()`, 203
rate of change, 223
ratio, coin toss, 19
ratio of posteriors, 158
  posterior odds, 159, 170–171, 177
reasoning, 70, 72, 73. *See also* Bayesian reasoning
reciprocal, multiplying, 106
`rnorm`, 207
RStudio, 196
`runif()`, 206–207

S

`sample()`, 207–208
`seq()`, 186, 204–205
`set.seed()`, 208–209
sigma events, 120
skepticism, 175–176
spatial reasoning, 73
spread, 103–106, 108
standard deviation, 103
  calculating, 112
  determining, 107–108
  formula, 108
  parameter in normal distribution, 111, 114
  usefulness, 108–109, 111
statistical analysis, 84–85
statistical reasoning, 8, 85
statistics. *See also* inference
  conditional probabilities, importance of, 60 (*see also* conditional probabilities)
  probability, *versus,* 46
stock price simulation, 213–214
sum rule for non-mutually exclusive events, 28–30, 61
`sum_then_square()`, 209–210
summation symbol, 42

T

true value, 105
Twilight Zone example, 168-172, 175–176

U

uncertainty, measuring, 13–14

V

variance, 103
  finding, 106–107
  properties, 107
  squaring, 107, 109
vectors
  `length()`, 202–203
  plots, as part of, 210
  R, 200–202
  stock price simulation, 213–214
  `sum()`, 203
villain bomb example, 112–114, 116–118, 120

W

weighted sums, 98