CONTENTS

PREFACE xi

PROLOGUE:
WHAT IS A FUNCTION? 1

Exercise 14

1
LET'S DIFFERENTIATE A FUNCTION! 15

Approximating with Functions 16
Calculating the Relative Error 27
The Derivative in Action! 32
Step 1 34
Step 2 34
Step 3 35
Calculating the Derivative 39
Calculating the Derivative of a Constant, Linear, or Quadratic Function 40
Summary 40
Exercises 41

2
LET'S LEARN DIFFERENTIATION TECHNIQUES! 43

The Sum Rule of Differentiation 48
The Product Rule of Differentiation 53
Differentiating Polynomials 62
Finding Maxima and Minima 64
Using the Mean Value Theorem 72
Using the Quotient Rule of Differentiation 74
Calculating Derivatives of Composite Functions 75
Calculating Derivatives of Inverse Functions 75
Exercises 76

3
LET'S INTEGRATE A FUNCTION! 77

Illustrating the Fundamental Theorem of Calculus 82
Step 1—When the Density Is Constant 83
Step 2—When the Density Changes Stepwise 84
Step 3—When the Density Changes Continuously 85
Step 4—Review of the Imitating Linear Function 88
Step 5—Approximation → Exact Value 89
Step 6—$p(x)$ Is the Derivative of $q(x)$ 90

The Manga Guide to Calculus
(C) 2009 by Hiroyuki Kojima, Shin Togami, and Becom Co., Ltd.

Using the Fundamental Theorem of Calculus 91
Summary 93
A Strict Explanation of Step 5 94
Using Integral Formulas 95
Applying the Fundamental Theorem 101
Supply Curve 102
Demand Curve 103
Review of the Fundamental Theorem of Calculus 110
Formula of the Substitution Rule of Integration 111
The Power Rule of Integration 112
Exercises 113

4
LET'S LEARN INTEGRATION TECHNIQUES! 115

Using Trigonometric Functions 116
Using Integrals with Trigonometric Functions 125
Using Exponential and Logarithmic Functions 131
Generalizing Exponential and Logarithmic Functions 135
Summary of Exponential and Logarithmic Functions 140
More Applications of the Fundamental Theorem 142
Integration by Parts 143
Exercises 144

5
LET'S LEARN ABOUT TAYLOR EXPANSIONS! 145

Imitating with Polynomials 147
How to Obtain a Taylor Expansion 155
Taylor Expansion of Various Functions 160
What Does Taylor Expansion Tell Us? 161
Exercises 178

6
LET'S LEARN ABOUT PARTIAL DIFFERENTIATION! 179

What Are Multivariable Functions? 180
The Basics of Variable Linear Functions 184
Partial Differentiation 191
Definition of Partial Differentiation 196
Total Differentials 197
Conditions for Extrema 199
Applying Partial Differentiation to Economics 202
The Chain Rule 206
Derivatives of Implicit Functions 218
Exercises 218

The Manga Guide to Calculus
(C) 2009 by Hiroyuki Kojima, Shin Togami, and Becom Co., Ltd.

EPILOGUE:
WHAT IS MATHEMATICS FOR? 219

A
SOLUTIONS TO EXERCISES 225

Prologue 225
Chapter 1 225
Chapter 2 225
Chapter 3 226
Chapter 4 227
Chapter 5 228
Chapter 6 229

B
MAIN FORMULAS, THEOREMS, AND FUNCTIONS
COVERED IN THIS BOOK 231

Linear Equations (Linear Functions) 231
Differentiation 231
Derivatives of Popular Functions 232
Integrals 233
Taylor Expansion 234
Partial Derivatives 234

INDEX 235

The Manga Guide to Calculus
(C) 2009 by Hiroyuki Kojima, Shin Togami, and Becom Co., Ltd.