

# INDEX

## Numbers

2-3 trees, 304, 311–312. *See also* B-trees

## A

abstract data types (ADTs), 37–47  
abstraction, 39  
creators, 40  
implementing, 40–46  
    with classes, 41  
    with functions, 43, 45  
mutations, 39–40  
mutators, 40, 44–45  
observers, 40  
operations, 39–40  
producers, 40  
abstraction, 38, 39  
adaptive sorting, 92  
addressable heap, 346  
Adelson-Velsky, Georgy, 249  
ADTs (abstract data types), 37–47  
algorithms  
    analysis of, 47, 50, 52, 55  
    backtracking, 69–71, 89  
    brute-force, 82  
    complexity, 50, 52  
    design, 63  
    divide-and-conquer, 65–68, 72  
    performance, 50–58  
amortized time performance, 54  
archery puzzle, 89  
assured balance binary search  
    trees, 249  
asymptotic notations, 51  
average-case performance of  
    algorithms, 54  
AVL trees, 235, 249–254, 261  
    adding to, 251  
    creating, 250

number of nodes, 280  
performance, 255  
removing from, 251  
rotation, 252

## B

Babel, 13, 19  
backtracking, 69, 70, 71, 89  
bags, 40–47, 203–205, 219–220,  
    232–233  
binary search trees, implementing  
    with, 239  
lists, implementing with, 207  
operations on, 204  
Bayer, Rudolf, 291  
BB[ $\alpha$ ] trees. *See* weight-balanced  
    bounded trees  
Bellman-Ford shortest path  
    algorithm, 466  
best-case performance of  
    algorithms, 54  
bidirectional bubble sort, 99  
Bierce, Ambrose, 218  
big Omega notation, 51  
big  $O$  notation, 51–59  
big Theta notation, 51  
binary heap, 318, 340, 341  
binary search, 56–57, 59, 166–168, 172  
binary search trees, 239–282, 485  
    adding values to, 241–242  
    assured balance, 249–261  
    AVL trees, 249–254, 261  
    bag, implementing, 239  
    balanced, 249–250  
    deleting from, 485  
    finding the maximum, 244–245  
    inorder traversal, 246–248, 280  
    operations on sets, 239  
    performance, 248–249

- binary search trees (*continued*)
- postorder traversal, 246, 280
  - preorder traversal, 246, 279–280
  - probabilistically balanced, 249, 261–278, 281–282
  - randomized binary search trees, 262–270, 281
  - rebalancing, 282
  - red-black trees, 304–314
  - removing values from, 242–244
  - searching in, 239–241
  - self-adjusting trees, 249, 261–278, 281–282
  - set, implementing, 239
  - splay trees, 270–278, 281–282
  - traversing, 246–248, 280
  - weight-bounded balanced trees, 255–261, 281
- binary trees, 237–282, 287, 288, 304
- 2-3 trees, 304, 311–312
  - complete, 238, 244, 247, 279
  - copying, 279
  - full, 237–238, 279, 280
  - heap-ordered, 347
  - height, 237, 238, 249–257, 279, 280
  - perfect, 238, 248, 256, 279, 282
  - size, 279
- binomial heaps, 351–367, 385
- adding values to, 354–356
  - changing values in, 360–362
  - implementing, 353–354
  - merging, 356–358
  - performance, 362–363
  - removing values from, 358–360
- binomial trees, 351–352
- bitmaps, 206
- bitmap selection, 122–123
- bitmap sort, 112–113
- bogosort, 117
- brute-force algorithms, 82–86, 87
- B-trees, 291–303, 315. *See also* red-black trees
- adding keys to, 295–298
  - implementing, 292–293
  - inorder traversal, 294–295
  - optimization, 315
  - performance, 303
  - removing keys from, 298–303
- searching in, 293–294
- traversing, 294–295
- bubble sort, 97–98, 100, 103–104, 118
- Burton, F. Warren, 470
- ## C
- chaining, hashing with, 219–221
- change, making, 87
- circular lists, 195–200
- adding elements to, 197
  - implementing, 198–199
  - joining, 200
  - operations on, 196
  - performance, 199–200
  - removing elements from, 197–199
- cocktail shaker sort (shuttle sort), 99–100
- coin tossing shuffling, 140–141
- comb sort, 103–104
- complexity of algorithms, 50–58
- connectivity detection, 428, 453–458
- search-based algorithm, 456–458
  - sets-based algorithm, 454–456
- counting selection, 123–124
- counting sort, 114–115
- cryptarithmic puzzles (cryptarithms), 83–86, 90
- cycle detection, 427, 452–453
- Tarjan’s algorithm, 448, 452–453
- ## D
- d-ary heaps, 340–341
- data structures
- binary search trees, 239–282, 485
  - deques, 191–195, 201
  - hash tables. *See* hashing
  - heaps. *See* heaps
  - lists, 177–184, 195–200, 201
  - queues, 188–191, 201, 326–327, 342, 346–347, 531
  - stacks, 184–188, 200, 201
  - treaps, 332–340, 343
  - trees. *See* trees
  - tries. *See* tries
- declarative-style programming, 26–30
- deques (double-ended queues), 191–195, 201
- adding elements to, 192–194
  - implementing, 194–195

operations on, 192  
performance, 195  
removing elements from, 193

Dijkstra's algorithm, 438  
    simplifying, 466

divide-and-conquer, 65–68, 72

double hashing, 226–229

double hashing with prime lengths, 229–232

doubling search. *See* exponential search

doubly linked lists. *See* circular lists

Dutch National Flag Problem, 119

dynamic programming (DP), 63, 72–82, 89–90  
    bottom-up, 72, 79–82  
    calculating Fibonacci series, 72–74, 79–80  
    line breaking, 74–79, 89  
    memoization, 72–74, 78–81  
    summing ranges, 80–82, 90  
    tabulation, 72  
    top-down, 72–79

## E

ECMAScript, 6, 8, 10–11, 13

ESLint, 18, 19

exponential search, 168–169, 173

extended heaps  
    binomial heaps, 351–363, 385  
    Fibonacci heaps, 367–376, 386  
    lazy binomial heaps, 363–367  
    pairing heaps, 376–384  
    skew heaps, 347–351, 385

external sorting, 92

## F

factorial, 52, 53, 65–66, 82, 88

Fibonacci heaps, 367–376, 386  
    adding values to, 371  
    changing values in, 372–375  
    implementing, 368–369  
    merging, 370–371  
    performance, 375–376  
    removing values from, 371–372

Fibonacci series, 66–67, 72–74, 79–80, 89  
    calculating with bottom-up DP, 79–80

calculating with top-down DP, 72–74

Fira Code Font, 15–16

Fisher-Yates sampling, 151–152

Fisher-Yates shuffling, 145–146, 151–152, 156

Flow type checker, 18

Floyd, Robert, 142, 329

Floyd's sampling, 148–150

Floyd's shuffling, 142–143

Floyd-Warshall algorithm, 430–434

forests, implementing, 288

FORTH, 185

FORTRAN, 24

functional data structures, 470–484  
    arrays, 470  
    binary search trees, 478–481  
    common lists, 470–473  
    Fibonacci heap, 481  
    hash tables, 470  
    queues, 474–478  
    stacks, 473–474

functional programming (FP)  
    declarative style, 26–30  
    higher-order functions, 30–32  
    impure functions, 33–34  
    reasons for using, 24  
    side effects, 32–33

## G

galloping search. *See* exponential search

general trees, 237

graphs  
    adjacency lists representation, 429–430  
    adjacency matrix representation, 428–429  
    adjacency set representation, 430  
    arcs, 425  
    arrows, 425  
    Bellman-Ford algorithm, 434–438  
    connectivity detection, 428, 453–458  
    cycle detection, 427, 452–453  
    defined, 425–427  
    degree of, 425  
    Dijkstra's algorithm, 438–444  
    edges, 425  
    Floyd-Warshall's algorithm, 430–434

graphs (*continued*)

- Hamiltonian cycle, 88
  - Kahn's algorithm, 445–448
  - Kruskal's algorithm, 88, 462–465
  - links, 425
  - minimum spanning trees, 88, 427, 458–465
  - neighbor, 425
  - nodes, 425
  - points, 425
  - Prim's algorithm, 459–462
  - representing, 428–430
  - shortest path problem, 427, 430–443, 466
  - sorting, 444–452
  - Tarjan's algorithm, 448, 452–453
  - topological sort, 427, 445–448, 451–452, 466
  - vertices, 425
- greedy algorithms, 63, 87–88

## H

- hashing, 218–232, 233
  - adding values to, 220, 224, 228, 231
  - chaining, with, 219–221
  - collision, 219, 222
  - creating, 220, 223–224, 227, 230–231
  - double hashing, 226–229
  - double hashing with prime lengths, 229–232
  - hash remainder function, 218
  - load factor, 222, 223, 225, 226, 228
  - open addressing, with, 221–226
  - performance, 221, 225
  - removing values from, 221, 225, 229, 232
  - resizing, 233
  - searching in, 220, 224, 228, 231
- heaps
  - adding to, 321–323
  - addressable heaps, 346–347
  - binary heaps, 318–325
  - binomial heaps, 351–363, 385
  - d-ary heaps, 340–341
  - Fibonacci heaps, 367–376, 386
  - Fredman, Michael, 368
  - heap-ordered binary trees.  
*See* skew heaps

heap property, 318, 319–320, 330,

332–337

implementing, 320–325

lazy binomial heaps, 363–367

leftist heaps, 347

max heap, 319

min heap, 319

operations, 320

pairing heaps, 376–384

performance, 325

priority queues, 326–327, 342

quaternary heaps, 340

removing from, 323–325, 342

searching, 342, 386

skew heaps, 347–350, 385

structure property, 318–319, 332

treaps, 332–340, 343

trinary heaps, 340

heapsort, 320, 327–331, 342, 343

analysis, 329

Floyd's heap-building enhancement, 329

Williams' original heapsort, 327

higher-order functions, 30–32

Hindley-Milner type system, 41

## I

immutability. *See* functional data structures

impure functions

avoiding state, 33–34

using injection, 34

inefficient sorting, 116–117

infinite loop. *See* infinite loop

inorder traversal of binary search trees, 246–248, 280

in-place sorting, 93

insertion sort, 101–103, 104, 105, 108, 111, 118

internal sorting, 92

interpolation search, 169–171

## J

JavaScript, 3–21, 25–34. *See also*

functional program-

ming (FP)

arrow functions, 4–5

classes, 5–6

closures, 11–13  
CommonJS modules, 9  
destructuring, 7–8, 9–10  
development tools, 13–20  
ECMAScript modules, 10–11  
filtering arrays, 27  
as a functional language, 25–34  
.indexOf method, 160  
looping through arrays, 30  
modules, 8–11, 13  
.pop method, 185, 200  
.push method, 185, 200  
.random method, 138  
reducing arrays to a value, 29–30  
searching arrays, 27–28  
sleep sort, 117  
.sort method, 95–96  
spread operator, 6–7  
testing arrays, 28, 36  
transforming arrays, 28–29, 35  
JSDoc, 16–18  
jump search, 163–166, 172

## K

Kahn’s algorithm, 445–448  
Knuth, Donald, 52, 152, 157  
Knuth’s sampling, 152  
Kruskal’s algorithm, 462–465

## L

Landis, Evgenii, 249  
lazy binomial heaps, 363–367  
adding values to, 364–365  
changing values in, 366–367  
implementing, 363–364  
performance, 367  
removing values from, 365–366  
lazy selecting, 132–134  
leftist heap, 347  
Lehmer code, 144  
linear search, 160–162  
LISP, 24, 177  
lists  
adding values to, 182  
appending to, 200  
circular lists, 195–200  
creating, 182  
deques, 191–195, 201

getting values at a position, 183  
implementing with arrays, 179–180  
implementing with dynamic  
memory, 180–183  
operations, 178  
ordered lists, 207–210  
performance, 184  
queues, 188–191, 201  
removing values from, 182–183  
searching in, 183–184  
self-organizing, 215–218, 232  
skip lists, 210–215, 232  
stacks, 184–188, 200, 201  
lottery drawing sampling, 150–151  
Loyd, Sam, 70, 89  
Lucas, Édouard, 67

## M

maps, 203, 205, 220, 221  
defined, 203  
operations, 205  
max heap, 319  
maze, solving, 69–70  
McCreight, Edward, 291  
median of medians selection,  
127–130  
meldable priority queues (MPQs),  
346–347

operations, 347

merge sort, 93, 110–112, 119, 133  
min heap, 319  
minimum spanning trees, 88, 427,  
458–465  
Kruskal’s algorithm, 462–465  
Prim’s algorithm, 459–462  
Mozilla Developer Network  
(MDN), 13  
multiset, 40–47  
mutual recursion. *See* recursion,  
mutual

## O

object-based tries, 401–405  
adding keys to, 404–405  
implementing, 402  
performance, 406  
removing keys from, 405–406  
searching, 402–404

- object-oriented programming (OOP), 23, 32
- offline sorting, 93
- O’Neill, Melissa, 470
- online sorting, 93
- only one value sampling, 146–147
- open addressing, hashing with, 221–226
- orchards, 284
- ordered lists, 207–210
- adding values to, 208–209
  - performance, 210
  - removing values from, 209–210
  - searching in, 207–208
  - sentinels and, 211, 212, 232
- out-of-place sorting, 93
- P**
- pairing heaps, 376–384
- adding values to, 378
  - changing values in, 382–384
  - implementing, 377
  - melding (merging), 377–378
  - performance, 384
  - removing values from, 378–381
- Pandita, Narayana, 84
- partition-exchange sort. *See* quicksort
- Pascal’s triangle, 352
- performance, algorithm, 50–59
- amortized time, 54
  - average case, 54
  - best case, 52, 54, 59
  - big Omega notation, 51–52
  - big  $O$  notation, 51–52, 57, 59
  - big Theta notation, 51, 59
  - classes, complexity, 52–54
  - constant order, 52, 53, 54
  - exponential order, 52, 53
  - factorial order, 52, 53
  - linear order, 52, 53, 55
  - logarithmic order, 52, 53
  - log-linear order, 52
  - measurements for, 54–55
  - quadratic order, 52, 53
  - small omega notation, 51, 52, 57
  - small  $o$  notation, 51, 52, 59
  - trade-offs, 57–58
  - worst case, 52, 54, 55, 57, 59
- performance measurements for algorithms, 54–55
- performance trade-offs, 57–58
- permutation, generating next, 84–86
- permutation sort, 117
- persistent data structures. *See* functional data structures
- PostScript, 185
- Prettier formatting, 16
- Prim’s algorithm, 459–462
- priority queues, 326–327, 342, 346–347, 531
- addressable, 346–347
  - addressable heaps, 346
  - leftist heap, 347
  - meldable, 346–347
  - operations, 326, 347
- probabilistic balance binary search trees, 249, 261–278, 281–282
- Q**
- quaternary heaps, 340
- queues, 188–191, 201, 326–327, 342, 346–347, 531
- adding elements to (queueing), 189
  - implementing, 189–191
  - operations, 188
  - performance, 191
  - priority, 326–327, 342, 346–347, 531
  - removing elements from (dequeueing), 189–190
- quickselect, 125–132, 135
- quicksort, 105–110, 119, 125–126, 127, 247, 264
- dual-pivot selection, 108–110
  - hybrid method, 107–108
  - “median of three” pivot selection, 107
  - pivot selection techniques, 106–107
  - standard version, 105–106
- R**
- radix sort, 112, 115–116, 119
- radix trees, 406–413
- adding keys to, 410–411, 424
  - creating, 407
  - performance, 414

- removing keys from, 412–414
  - searching, 407–409
  - randomized binary search trees, 262–270, 281
    - adding keys to, 263–264
    - creating, 262–263
    - joining, 268–269
    - performance, 269–270
    - removing keys from, 267–268
    - splitting, 264–267
  - random number generation, 138
  - React Redux, 40
  - recursion, 64–72, 82, 89, 208
    - divide-and-conquer, 65–68, 72
    - memoization, 73–74, 78–81
    - mutual. *See* mutual recursion
  - red-black trees, 304–314
    - adding keys to, 306–307
    - implementing, 305–306
    - performance, 313–314
    - removing keys from, 309–313
    - restoring structure of, 307–309
  - repeated step pivot selection, 130–132
  - reservoir sampling, 153–154
  - reverse Polish notation (RPN), 185
  - Robson, J. M., 143
  - Robson’s algorithm, 143–145, 156
- S**
- sampling, 146–153, 156, 157
    - Fisher-Yates algorithm, 151–152
    - Floyd’s algorithm, 148–150
    - Knuth’s algorithm, 152–153
    - lottery drawing, 150–151
    - only one value, 146–147
    - with repetition, 146–147, 156
    - without repetition, 147–154
    - reservoir sampling, 153–154
    - several values, 147
    - by sorting or shuffling, 148
  - search
    - binary search, 56–57, 59, 166–168, 172
    - definition, 159–160
    - exponential search, 168–169, 173
    - interpolation search, 169–171
    - jump search, 163–166, 172
  - linear search, 160–161
  - linear search with sentinels, 162–163
  - ordered arrays, 163–171
  - unsorted arrays, 160–162
  - Sedgewick, Robert, 304, 312
  - selecting
    - bitmap selection, 122–123
    - counting selection, 123–124
    - lazy select, 132–134
    - median of medians pivot selection, 127–130
    - merge sort, 133
    - quickselect, 125–132, 135
    - quicksort, 105–110, 119, 125–126, 127, 247, 264
    - repeated step pivot selection, 130, 131, 135
    - selecting with comparisons, 124–125, 130
    - selecting without comparisons, 122–124
    - selection sort, 100–101, 124–125, 135
    - self-adjusting trees, 249, 261–278, 281–282
    - self-organizing lists, 215–218, 232
      - adding values to, 217
      - count ordering strategy, 218
      - move to front (MTF) strategy, 218
      - performance, 217–218
      - removing values from, 217
      - searching in, 215–217
      - swap with previous strategy, 218
      - variants, 217–218
    - sets
      - binary search trees, implementation with, 239–282
      - bitmaps, implementation with, 206
      - hashed, 233
      - JavaScript objects, with, 205–206
      - lists, implementation with, 207–218
      - operations, 204
    - Shell sort, 104–105

- shortest path problem, 427, 430–443, 466
  - Bellman-Ford algorithm, 434–438
  - Dijkstra’s algorithm, 438–444
  - Floyd-Warshall’s algorithm, 430–434
- shuffling, 139–146, 148, 151, 155–157
  - by coin tossing, 140–141
  - Fisher-Yates algorithm, 145–146, 151–152, 156
  - Floyd’s algorithm, 142–143
  - in linear time, 142–146
  - permutation, 139, 142–146, 155
  - Robson’s algorithm, 143–145, 156
  - by sorting, 139–140
- shuttle sort, 99–100
- side effects
  - keeping inner state, 32
  - mutating arguments, 33
  - returning impure functions, 33
  - using global state, 32
- sinking sort, 99, 118
- skew heaps, 347–350, 385
  - adding keys to, 350
  - implementing, 348
  - merging, 348–350
  - performance, 350–351
  - removing keys from, 350
- skip lists, 210–215, 232
  - adding values to, 213–214
  - creating, 211–212
  - performance, 215
  - removing values from, 214–215
  - restructuring, 232
  - searching in, 212
- sleep sort, 117
- slow sort, 116
- small omega notation, 51, 52, 57
- small *o* notation, 51, 52, 59
- sorting
  - adaptive, 92–93
  - bidirectional bubble sort, 99
  - bitmap sort, 112–113, 115
  - bogosort, 117
  - bubble sort, 97–98, 100, 103–104, 118
  - cocktail shaker sort, 99
  - comb sort, 103–104
  - counting sort, 114–115
  - Dutch National Flag Problem, 119
  - external, 92
  - heapsort, 320, 327–331, 342, 343
  - inefficient, 116–117
  - in-place, 93
  - insertion, 108
  - insertion sort, 101–103, 104, 105, 108, 111, 118
  - internal, 92
  - merge sort, 93, 110–112, 119, 133
  - offline, 93
  - online, 93, 105
  - out-of-place, 93
  - partition-exchange sort, 105
  - performance, 93, 96–98, 101, 105–108, 110–112, 117–119
  - permutation sort, 117
  - preorder traversal of binary search trees, 280
  - quicksort, 105–110, 119, 125–126, 127, 247, 264
  - radix sort, 112, 115–116, 119
  - selection sort, 100–101, 124–125, 135
  - Shell sort, 104–105
  - shuttle sort, 99
  - sinking sort, 99, 118
  - sleep sort, 117
  - slow sort, 116
  - .sort method, 95–96
  - stability, 93–94, 96, 111, 116, 118
  - stooge sort, 116
  - Tim sort, 111
  - splay trees, 270–278, 281–282
    - adding keys to, 274–275
    - performance, 278
    - removing keys from, 275–278
    - searching in, 274
    - splaying a tree, 271–274
  - Squarest Game on the Beach, 70, 89
  - stacks, 184–188, 200, 201
    - adding to (pushing), 185–187
    - implementing with arrays, 185
    - operations, 185
    - performance, 188
    - popping (removing from), 185–187

Stirling’s approximation, 97  
stooge sort, 116

## T

Tarjan, Robert, 368, 448, 452–453  
Tarjan’s algorithm, 448, 452–453  
tautologies, detecting, 82–83  
ternary heaps, 340–341  
ternary search tries, 414–424  
    adding keys to, 417–419  
    creating, 415–416  
    performance, 423  
    removing keys from, 419–423  
    searching in, 416–417  
    storing extra data in, 416  
Tim sort, 111  
topological sort, 427, 445–448,  
    451–452, 466  
    Kahn’s algorithm, 445  
    Tarjan’s algorithm, 448, 452–453  
Towers of Hanoi, 67, 89  
traveling salesman problem, 84, 87–88  
treaps, 332–340, 343  
    adding keys to, 333–336  
    creating, 332–333  
    merging, 343  
    performance, 339–340, 344  
    removing keys from, 336–339, 343  
    searching, 332–333  
    splitting, 343  
trees  
    2-3 trees, 304, 312  
    adding nodes to, 286  
    ancestors, defined, 237  
    AVL trees, 235, 249–254, 261  
    binary search trees, 239–282, 485  
    binary trees, 237–282, 287, 288, 304  
    binomial trees, 351–352  
    breadth-first (level-order)  
        traversal, 290–291  
    B-trees, 291–303, 315  
    children, defined, 237  
    defining, 283–290  
    degree, 237, 284  
    depth-first traversal, 289–290  
    descendants, defined, 237  
    equality, 314–315  
forests, 284, 288  
general trees, 237  
groves, defined, 284  
height, 237, 272  
implementing with arrays, 284–287  
implementing with binary trees, 287  
inorder traversal, 289, 294  
level, defined, 237  
minimum spanning trees, 88, 427,  
    458–465  
multiary (multiway), defined, 284  
orchards, defined, 284  
ordered forests, 284  
organigram (organizational  
    chart), 236  
parent, defined, 237  
postorder traversal, 289, 314  
preorder traversal, 289, 314  
probabilistic balance binary  
    search trees, 249,  
        261–278, 281–282  
radix trees, 406–413  
red-black trees, 304–314  
removing nodes from, 286–287  
representing, 237, 287–291  
roots, defined, 237  
size, defined, 237  
splay trees, 270–278, 281–282  
traversing, 288–291  
weight-bounded balanced trees,  
    255–261, 281

## tries

    adding keys to, 394–397, 404–405,  
        410–411, 417–419  
    classic, 388–401, 424  
    implementing, 388–390, 402, 407,  
        415–416  
    object-based, 401–406  
    performance, 401, 406, 414, 423  
    radix trees, 406–413  
    removing keys from, 397–401,  
        405–406, 412–413,  
        419–423  
    searching in, 390, 402–403,  
        407–409, 416–417  
    storing extra data in, 390–406  
    ternary search tries, 414–423

trinary heaps, 340

TypeScript, 18–19

## V

Visual Studio Code (VSC), 4,  
13–15, 18

## W

Waterman, Alan, 153

WebAssembly (WASM), 185

weight-bounded balance (BB[ $\alpha$ ]) trees,

255–261, 281

adding keys to, 257

creating, 256–257

finding by rank, 260–261

fixing balance, 257–259

performance, 261

removing keys from, 257

Williams, John W. J., 327–329

worst-case performance of algorithms, 54