MATHEMATICAL FUNCTIONS

Scratch supports a large number of mathematical functions. This appendix provides a quick review of these functions with some practical examples of their uses.

Math Functions in Scratch

The **sqrt of** block from the *Operators* palette contains 14 math functions that you can choose from its drop-down menu. Table 1 briefly describes these functions.

Table 1: Scratch's Mathematical Functions

Function	Description
	Returns the absolute value of *x*. For example, *abs*(5) = 5, *abs*(0) = 0, and *abs*(–4) = 4. Geometrically, *abs*(*x*) is the distance between *x* and 0 on the number line. Similarly, *abs*(*x* – *y*) is the distance between *x* and *y* on the on the number line.
	Returns the largest integer that is less than or equal to *x*. For example, *floor*(2.1) = 2, *floor*(2.9) = 2, *floor*(–2.1) = –3.
	Returns the smallest integer that is greater than or equal to *x*. For example, *ceiling*(2.1) = 3, *ceiling*(2.9) = 9, *ceiling*(–2.1) = –2.
	Returns the square root of *x*. This is another number *y* such that $y^2 = x$. For example, *sqrt*(16) = 4, *sqrt*(2) = 1.4142, and *sqrt*(0) = 0. Passing a negative value for *x* returns NaN (short for "not a number").
	Returns the sine of *x*, where *x* is an angle expressed in degrees. For example, *sin*(0) = 0, *sin*(30) = 0.5, and *sin*(90) = 1.
	Returns the cosine of *x*, where *x* is an angle expressed in degrees. For example, *cos*(0) = 0, *cos*(60) = 0.5, and *cos*(90) = 0.
	Returns the tangent of *x*, where *x* is an angle expressed in degrees. For example, *tan*(0) = 0, and *tan*(45) = 1.
	Returns the inverse sine, or arcsine, of *x*. The arcsine of *x* is the angle whose sine is *x*. For example, *asin*(0.5) = 30.
	Returns the inverse cosine, or arccosine, of *x*. The arccosine of *x* is the angle whose cosine is *x*. For example, *acos*(0.5) = 60.
	Returns the inverse tangent, or arctan, of *x*. The arctan of *x* is the angle whose tangent is *x*. For example, *atan*(1) = 45.
	Returns the natural logarithm of *x*. For example, *ln*(2.718) ≈ 1.
	Returns the base-10 logarithm of *x*. For example, *log*(1000) = 3.
	Returns the exponential function of *x*. For example, $e^1 \approx 2.718$.
	Returns 10 to the power of *x*. For example, $10^2 = 100$.

In the following sections, we'll talk about some of these functions in more detail.

Trigonometric Functions

The basic trigonometric functions of an acute angle (one smaller than 90°) in terms of the sides of a right triangle are defined in Figure 1. These functions are simply names given to ratios that can be formed by the three sides of a right triangle.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

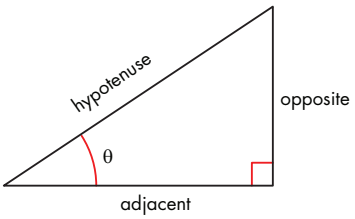

Figure 1: Definitions of the basic trigonometric ratios

As an example, let's say that we want to find the length of the shadow cast by a building 50 meters high when the sun is 20 degrees above the horizon, as illustrated in Figure 2.

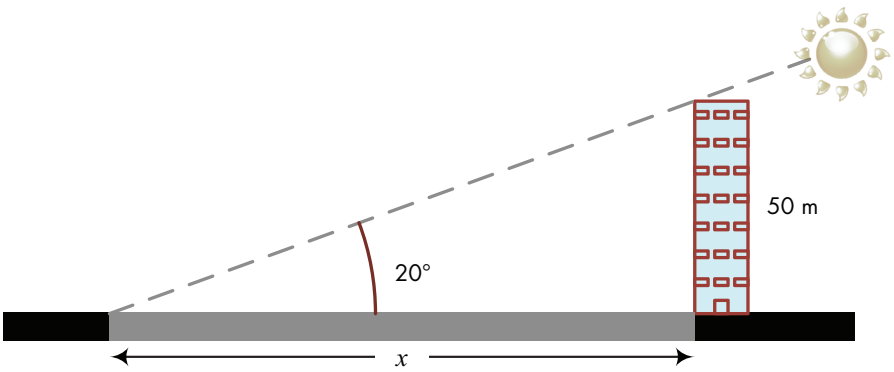

Figure 2: We can use trigonometry to find the length of a cast shadow.

Using the above definitions, we can write:

$$\tan(20) = \frac{50}{x}$$

From this, we can isolate x:

$$x = \frac{50}{\tan 20}$$

We can then calculate and display x using the following command:

The block returns 137.37 meters.

The inverse trigonometric functions, also known as the arc functions, are simply the reverse of the trigonometric functions. The arcsine of x is the angle whose sine is x. Similarly, the arccosine of x is the angle whose cosine is x, and the arctan of x is the angle whose tangent is x.

Logarithmic Functions

The base-10 logarithm, $\log(x)$, is the inverse function of 10^x. That is, if $\log(x) = y$, then $x = 10^y$. An example of the use of the base-10 logarithm is the decibel (or dB) scale, which measures the loudness of sound. If I is the intensity of sound in watts per square meter, then the sound level in decibels is given by the following:

$$\text{Sound level} = 10 \log\left(I \times 10^{12}\right) \text{ dB}$$

For example, if the sound intensity is 10^{-10} watts per square meter, we can find the sound level in dB using the following blocks:

Executing these blocks gives an answer of 20 dB.

Scratch also has a command to calculate the natural logarithm of a number. The natural logarithm of a positive number x, denoted by $\ln(x)$, is defined in terms of the area under the curve $y = 1/x$. The function is not defined for $x \leq 0$.

As a practical example, let's consider the problem of finding the half-life of a radioactive material, which is defined as the time required for half of the radioactive atoms in a sample to decay. The formula for calculating half-life is

$$\text{Half-life} = \frac{\ln(2)}{k},$$

where k is a constant that depends of the radioactive material. Polonium-210, for example, has the constant $k = 0.005$. Using Scratch, we can find the half-life of polonium-210 as follows:

This command gives an answer of about 139 days.

The number e is the value that satisfies $\ln(e) = 1$, and the exponential function, $\exp(x) = e^x$, is the inverse of $\ln(x)$. That is, if $\ln(x) = y$, then $x = e^y$. This function appears in many scientific formulas such as those modeling population growth, compound interest, radioactive decay, heat transfer, and so on.

As an example, let's assume that the number of people in a city with an initial population of 1,000 is expected to grow according to this formula:

$$\text{Number after } t \text{ years} = 1{,}000e^{0.05t}$$

To find the number of people after 10 years, we can re-create the formula with Scratch commands and replace t with 10:

This gives an answer of about 1,649.

Note that Scratch does not have a built-in function for calculating x^y. However, you can create this function by noting that

$$x^y = e^{y \ln x}, \quad x > 0.$$

For example, if you want to find 6^4, you can use the following block:

This block gives the answer of 1,296.